2/27/20

Module 2: Frequency Models

Section 1: Basics

N = rvr the number of losses/accidents/claims per "exposure unit" ($N = 0, 1, 2, \ldots$)

An exposure unit may be a time unit (e.g. years, months, etc) or some other unit of measurement (e.g. 10000 miles driven)

Notation: $Pr(N=k) = p_k$

The probability generating function for N is defined by $P_N(z) = E[z^N]$

Note:

N	z^N	Pr
0	1	p_0
1	z	p_1
2	z^2	p_2
⋮	⋮	

$$\therefore E[z^N] = p_0 + p_1 \cdot z + p_2 \cdot z^2 + \cdots = P_N(z)$$

Remark: $P_N(0) = p_0$

$P_N'(0) = p_1$

$P_N''(0) = 2 \cdot p_2 \implies p_2 = \frac{P_N''(0)}{2!}$

$P_N'''(0) = 3! \cdot p_3 \implies p_3 = \frac{P_N'''(0)}{3!}$

Generally, $p_k = \frac{P_N^{(k)}(0)}{k!}$

After applying a deductible, let

M = r.v.r. the number of <u>payments</u> per exposure unit

Define $\pi = Pr(\text{a loss results in a payment}) = Pr(X > d)$

Define T_j = rvr the interevent exposure fraction $j = 0, 1, 2, \ldots$

E.g. T_0 = rvr the fraction of the exposure unit until the 1st loss

T_1 = ——————————————————— from the 1st loss until the 2nd loss

⋮

<u>Remark</u>: $\{T_j\}$ are ~~mutual~~ mutually independent

<u>Section 2</u>: $N \sim P(\lambda)$ Poisson Random Variable
(See P. 13 of Tables)

$$E[N] = \lambda = Var(N)$$

Partitions & Sums:

<u>Partitions</u>: $M \sim P(\lambda \cdot \pi)$

<u>Sums</u>: If N_1 & N_2 are independent with λ_1 & λ_2, resp.

then $N = N_1 + N_2 \sim P(\lambda_1 + \lambda_2)$

$\{T_j\}$:

$$T_0 \sim Exp\left(\theta = \frac{1}{\lambda}\right)$$

$$T_1 \sim Exp\left(\theta = \frac{1}{\lambda}\right)$$

$\{T_j\}$ are iid to $T \sim Exp\left(\theta = \frac{1}{\lambda}\right)$

Section 3: $N \sim NB(r, \beta)$ Negative Binomial R.V.

(See P. 14 of Tables)

$$E[N] = r \cdot \beta < Var(N) = r \cdot \beta(1+\beta)$$

Partitions & Sums

$$M \sim NB(r, \pi \cdot \beta)$$

$$N_1 \sim NB(r_1, \beta) \quad \& \quad N_2 \sim NB(r_2, \beta) \quad (\text{same } \beta)$$

$$\xrightarrow{\text{ind.}} N = N_1 + N_2 \sim NB(r_1 + r_2, \beta)$$

$$T_0 \sim Exp\left(\Theta = \frac{1}{r \ln(1+\beta)}\right)$$

$$T_1 \sim Exp\left(\Theta = \frac{1}{(r+1) \cdot \ln(1+\beta)}\right)$$

$$\vdots$$

$$T_j \sim Exp\left(\Theta = \frac{1}{(r+j) \cdot \ln(1+\beta)}\right)$$

Remarks:

1) If $r = 1$, we get a Geometric Distribution

$$NB(r=1, \beta) = Geom(\beta)$$

2) "Sums" implies that adding independent geometric distributions with the same β yields a negative binomial distribution

3) (As before)

$$N|\Lambda \sim P(\Lambda) \quad \& \quad \Lambda \sim \Gamma(\alpha, \Theta)$$

$$\Longrightarrow N \sim NB(r = \alpha, \beta = \Theta)$$

Section 4: $N \sim B(m, q)$ Binomial R.V.

(See P. 13 of Tables)

$E[N] = mq > Var(N) = mq(1-q)$

$M \sim B(m, \pi \cdot q)$

$N_1 \sim B(m_1, q)$ & $N_2 \sim B(m_2, q)$ (same q)

$\xRightarrow{ind} N = N_1 + N_2 \sim B(m_1 + m_2, q)$

$T_j \sim Exp\left(\Theta_j = \frac{1}{(j-m) \cdot \ln(1-q)}\right)$